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ARTICLE

THE CULTIVATION AND PRACTICE OF INNOVATIVE THINKING ABILITY IN HIGHER MATHEMATICS TEACHING

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ABSTRACT

Innovative thinking ability is one of the essential qualities of high quality applied talents, and it is especially important to improve students' innovative thinking ability in the process of teaching higher mathematics. The teaching process is dialectical and unified, and the teaching of mathematical thinking and mathematical concepts, concept recognition teaching method and independent inquiry teaching method can cultivate students' independent learning ability, stimulate their innovative thinking ability, and improve the teaching quality and learning effect of higher mathematics.

KEYWORDS

Higher Mathematics; Creative Thinking; Concept Identification; Independent Inquiry

1. INTRODUCTION

General Secretary Xi Jinping pointed out during his visit to Tsinghua University in 2021 that the catalyst of cross-fertilization of disciplines should be used to strengthen the cultivation ability of basic disciplines and enhance the original innovation ability. As a public compulsory general course for the quality training of talents in science and technology institutions, advanced mathematics is the theoretical foundation for the study of professional courses, an important tool for the analysis of scientific research work, and a necessary carrier for the cultivation of innovative thinking ability. Therefore, how to optimize teaching methods in the teaching process, expand the connotation education and stimulate students' innovative thinking ability is an important research content for the quality of talents cultivation in colleges and universities.

The higher mathematics course is characterized by theoretical, abstract, rigorous and logical features, which highlights the value goal and value direction of higher mathematics culture and education, but at the same time, the characteristics of the course determine that the classroom teaching atmosphere is less active than other courses. Ms. Zhu proposed a "2*3" teaching model to cultivate students' innovative thinking in university mathematics, which opens up a path for cultivating innovative thinking [1]. Wang and other teachers proposed to add strong teacher-student communication and discussion, infiltrate modern mathematical ideas in teaching and combine the use of mathematical software to enhance mathematical teaching thinking. Adhering to the goal of cultivating talents in colleges and universities where literacy-oriented ability is important, in the process of teaching higher mathematics, it is important to recognize that the understanding of mathematical concepts is crucial to the learning of higher mathematics, which should be used as the focus point to explore the connotative development and provide the source power to continuously promote the outbreak of innovative thinking [2]. Therefore, based on the teaching objectives of higher mathematics and the requirements of reform practice, adopting concept recognition and independent inquiry-based teaching methods can cultivate students' internal drive for active learning, encourage students' personalized development and stimulate their innovative thinking ability. The following are some practical cases to illustrate the theoretical ideas and specific practices of using concept recognition and independent inquiry teaching methods in the teaching process.

2. PERCEPTION AND TEACHING OF CONCEPT RECOGNITION-BASED PEDAGOGY

Concept recognition-based teaching method is a two-way interactive teaching mode for teachers and students that highlights the teaching of mathematical concepts. On the one hand, teachers are required to strengthen the deep processing of definitions and theorems in the teaching process, present them in a more intuitive and understandable form, and constantly strengthen students' understanding of the conceptual principles repeatedly reproduced; on the other hand, students are required to master the application of conceptual principles of knowledge points through repetitive special exercises in the learning process according to the specifics of course content.

2.1 Specific ideas of concept recognition-based pedagogy

In the teaching process of advanced mathematics, teachers take students as the main body, combine teaching objectives and teaching requirements, design representative cases as the entry point of concept teaching, inspire students to deeply understand concepts, accurately judge the theoretical principles required for problem solving, ask a series of questions in a multi-dimensional, deep and comprehensive way, and guide students to discuss the teaching contents. In the classroom, teachers can use various forms such as group discussions, teaching guidance and offline tutorials to mobilize students' desire for knowledge, expression and innovation in thinking, to continuously deepen their understanding of the same concept and enhance their ability to master the application of conceptual knowledge points [3]. The process of identifying conceptual principles is also the process of deepening understanding, discussing and extending the original concept, which not only can mobilize students' subjective initiative in learning, but also can guide students to explore the related knowledge system structure while understanding the concept and enhance their thinking and innovation ability.

2.2 Classroom examples of concept recognition-based teaching methods

The following example is a consolidation exercise after the first section on the definition of definite integrals. For such a problem of finding the limit sum

$$\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+L+\frac{1}{n}\right) \tag{1}$$

Extensive discussions and comprehensive analyses were conducted.

The students were first guided to observe the formal characteristics of the limit sum above: the expression is an n-term summation problem; this expression is encountered in similar expressions in which conceptual knowledge points.

Next, students were asked to discuss what knowledge points might be used to solve the problem. Many students could accurately identify the knowledge points based on the form of expressions which could be the two-sided clip criterion and the definition of definite integrals learned in Chapter 1. Then, students were asked to try with the two-sided clip criterion first and found that the conditions required to satisfy the two-sided clip criterion were not satisfied after enlargement and reduction, and then they tried to deal with it by applying the definition of definite integrals.

Analysis 1: Many students have the idea of using the "definition of a definite integral" step, keeping the constant 2 in 2n unchanged, first writing the $\sum$ form as a summation form, finding $\Delta x_i=\frac{1}{n}$, taking the point $\Delta\xi_i=\frac{i}{n}$, determining the interval [0,1], finding the function $f(x)=\frac{1}{1+2x}$, and thus having

$$\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+2\frac{i}{n}}=\int_0^2\frac{1}{1+2x}dx=\frac{1}{2}\ln 3 .$$

Analysis 2: Still use the steps of "definition of definite integral", keep 2n unchanged, first write $\sum$ form into summation form, find $\Delta x_i=\frac{2}{n}$, take the point $\Delta\xi_i=\frac{i}{n}$, determine the interval [0,2], find the function $f(x)=\frac{1}{1+x}$, thus we have

$$\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+2i}$$
$$=\frac{1}{2}\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+\frac{2i}{n}}=\frac{1}{2}\int_0^2\frac{1}{1+x}dx=\frac{1}{2}\ln 3$$

Further, on the basis of this limit and the topic, a simple deformation is made to the problem set. The deformed problem is

$$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+3}+\ldots+\frac{1}{n+(2n+1)}\right) \tag{2}$$

Ask students to continue the discussion and analysis by observing how the expression (2) is similar to (1), and what is the way to approach the problem for the changed limits and forms?

Analysis 3: Some students suggested that the problem could still be handled in the same way as the "Definite Integral Definition", but the last item was added and had to be handled separately, so that the middle point of each interval could be chosen. The problem is solved as follows: find $\Delta x_i=\frac{2}{n}$, determine the interval [0, 2] into n equal parts, and get n small intervals, $\left[0,\frac{2}{n}\right],\left[\frac{2}{n},\frac{4}{n}\right],\ldots,\left[\frac{2n-2}{n},\frac{2n}{n}\right]$, take the middle point of each small interval in turn, i.e., take the point $\Delta\xi_i=\frac{2i-1}{n}$, notice the last item to be listed separately according to the taking point, and determine the function $f(x)=\frac{1}{1+x}$, so as to have .

$$\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+(2n+1)}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+(2i+1)}$$
$$=\frac{1}{2}\lim_{n\to\infty}\frac{2}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{3}{n}}+\ldots+\frac{1}{1+\frac{(2n-1)}{n}}\right)+\frac{1}{2}\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\frac{1}{1+\frac{(2n+1)}{n}}$$
$$=\frac{1}{2}\int_0^2\frac{1}{1+x}dx=\frac{1}{2}\ln 3$$

Analysis 4: Some students also suggested that the two-sided clamping criterion can be combined with the result of the first question together, which can also solve this question, remembering $I_n=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+3}+\ldots+\frac{1}{n+(2n+1)}\right)$ then

$$I_n<\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}\right)$$
$$=\lim_{n\to\infty}\frac{1}{n}+\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}\right)=0+\frac{1}{2}\int_0^2\frac{1}{1+x}dx+\frac{1}{2}\ln 3$$
$$I_n>\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}+\frac{1}{n+2n+2}\right)$$
$$=\lim_{n\to\infty}\left(\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{n+2n}\right)+\lim_{n\to\infty}\frac{1}{n}=\frac{1}{2}\int_0^2\frac{1}{1+x}dx+0=\frac{1}{2}\ln 3$$

Analysis 5: Some students found another way to propose the use of Euler's constant definition, the content of the extrapolated textbook knowledge, combined with the two-sided clip criterion can also deal with the problem, the method is as follows

Because of $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ln n=C+o(1)$, where C is the Euler constant, so,

$$\frac{1}{n+1}+\frac{1}{n+3}+\ldots+\frac{1}{n+(2n+1)}=\ln\frac{3n+1}{n}+o(1)$$

$$I_n\le\lim_{n\to\infty}\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+2n}+\frac{1}{n+(2n+1)}\right)=\lim_{n\to\infty}\frac{1}{2}\ln\frac{3n+1}{n-1}=\frac{1}{2}\ln 3$$

$$I_n\ge\lim_{n\to\infty}\frac{1}{2}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+2n}+\frac{1}{n+(2n+1)}+\frac{1}{n+(2n+2)}\right)$$
$$=\lim_{n\to\infty}\frac{1}{2}\ln\frac{3n+2}{n}=\frac{1}{2}\ln 3$$

The flexibility of the concept identification method in the definition of definite integrals can be found in topic (1) and its variation (2). Several analytical methods listed in the previous section, although taking different points, having different forms of the product function and different integration intervals, yield the same results and the final conclusions remain the same. The teacher's task is to guide and help students to think about concepts, not to instill ready-made knowledge directly into them [4]. The teaching practice shows the importance of flexible use of concepts, which stimulates students' interest in asking questions about concepts and develops their creative thinking skills in using definitions, comparisons and extrapolations to solve problems.

3. COGNITION AND TEACHING OF SELF-DIRECTED INQUIRY-BASED PEDAGOGY

The teaching philosophy is based on both teachers and students as learners, discussing and researching new learning contents together, and focusing on cultivating students' innovative thinking skills in analyzing, solving and dispersing problems. Through the creation of inquiry problem content, guide students to independent inquiry, summarize the laws of inquiry, and continuously improve students' innovative thinking skills [5].

3.1 Specific ideas for self-directed inquiry-based teaching

According to the teaching content of each lesson, teachers carefully prepare example problems that match the knowledge points, are inspiring and extended, extend and adapt the examples from multiple visual perspectives, and guide students to think independently. Meanwhile, during the teaching process, we respect students' individual differences, advocate personalized development and diversified education, teach according to their abilities, and encourage students to study and learn independently. At least one open-ended exercise is assigned at the end of each course, so that students can analyze, adjust and adapt the original topic to derive a series of personalized topics of different difficulty levels. Combined with the teaching class information platform, such as QQ group and WeChat platform, the better adapted topics and solutions are selected for display and praised on the information platform to stimulate students' internal learning drive.

3.2 Self-directed inquiry-based teaching case one

Let's start with a teaching example of a function derivative problem.

After teaching the derivative function $y=x^x$ of the function $y=x^x(lnx+1)$, the teacher first leads students to consider what is the derivative form of $y=x^{g(x)}$, where the function g(x) is derivable, and after converting it into exponential form, combined with the derivative rule of the composite function, there is

$$y'=(x^{g(x)})'=(e^{g(x)\ln x})'=e^{g(x)\ln x}(g'(x)\ln x+g(x)\bullet\frac{1}{x})=x^{g(x)}(g'(x)\ln x+g(x)\frac{1}{x})$$

Extending this further, let the function f(x), g(x) all can lead, consider the derivative of y=fg. Similar to the previous analysis, it is not difficult to obtain

$$y'=(f^g)'=(e^{g\ln f})'=e^{g\ln f}(g'\ln f+g\bullet\frac{1}{f}\bullet f')=f^g\bullet(g'\ln f+g\bullet\frac{1}{f}\bullet f')$$

In addition to converting the function into an exponential function and using the composite function to derive the conclusion, students are led to actively consider that the original function can also be transformed into the form of a logarithmic function for processing by first taking the logarithm of both sides of the function at the same time, which can be obtained $\ln y=\ln f^g=g\ln f$.

Both sides of the equation are simultaneously derived with respect to the variables, then we have $\frac{1}{y}\bullet y'=g'\bullet\ln f+g\bullet\frac{1}{f}\bullet f'$ thereby having

$$y'=y(g'\bullet\ln f+g\bullet\frac{1}{f}\bullet f')=f^g(g'\bullet\ln f+g\bullet\frac{1}{f}\bullet f')$$

By exploring the original problem layer by layer, the students deepened their understanding of the derivative of such problems, improved their knowledge of the algorithm of derivatives, the use of the derivative of complex functions and the derivative of elementary functions, and deepened their understanding of the use of the derivative of implicit functions, and achieved the effectiveness of innovative thinking.

3.3 Example of self-directed inquiry-based teaching method II

In the following teaching example, the corresponding chain study is carried out for an integration problem.

Initial exploration: consideration $I_1=\int\frac{\cos x}{\sin x+\cos x}dx$.

According to the priority of the product function, the knowledge point is to use the universal formula, i.e., the commutative method, so that $t=\tan\frac{x}{2}$, thus converting it into the form of an indefinite integral of rational functions, which is a general teaching solution mode. Here is another angle to consider the indefinite integral of this topic, and observe that sin x and cos x are closely related, if the denominator of the product function is kept unchanged and the numerator is replaced by , that is $I_2=\int\frac{\sin x}{\sin x+\cos x}dx$.

So is there some intrinsic connection between the two? Adding I1 and I2, we get

$$I_1+I_2=\int\frac{\cos x}{\sin x+\cos x}dx+\int\frac{\sin x}{\sin x+\cos x}dx=x+C_1 \quad (3)$$

Subtract I1 and I2 from this to get

$$I_1-I_2=\int\frac{\cos x}{\sin x+\cos x}dx-\int\frac{\sin x}{\sin x+\cos x}dx \quad (4)$$
$$=\int\frac{\cos x-\sin x}{\sin x+\cos x}dx=\ln(\sin x+\cos x)+C_2$$

Combine (3) with (4) and solve the system of equations to obtain

$$I_1=\frac{1}{2}\left[x+\ln(\sin x+\cos x)\right]+C$$

where C represents the integration constant.

In addition, if we make a variable substitution $x=\frac{\pi}{2}-t$, there is also

$$\int\frac{\cos x}{\sin x+\cos x}dx=\int\frac{\sin t}{\sin t+\cos t}dt \ .$$

The result holds, inspired by the above conclusion, and then study the definite integral

$$I_1=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$$

can be directly substituted using equation (5) or as a variable substitution asd, easily obtained

$$I_1=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{\cos x+\sin x}{\sin x+\cos x}dx=\frac{\pi}{4}$$

4. CONCLUSION

In this paper, through teaching practice cases, we use concept identification-based teaching method and independent inquiry-based teaching method to analyze different theoretical ideas and specific practices to solve the same problem in a multi-dimensional, holistic and deep level. In terms of realistic classroom teaching effects, it can stimulate students' thinking and innovation ability, learn to explore the unknown from the known, discover the essence of the problem, and learn from one example to achieve twice the result with half the effort. Therefore, research-based teaching can not only enrich and deepen teaching content, expand and extend the amount of teaching information but also stimulate students' enthusiasm for independent learning and inquiry learning, improve students' research learning ability, and better promote the effectiveness of classroom teaching and the improvement of teaching quality [6].

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